РЕЗЮМЕ. Мақолада республикамиздаги олий таълим муассасалари, Илмий текшириш институтлар ва Олий аттестация комиссияси ўртасида фан ва таълимга оид электрон ҳужжат айланиш тизими жорий қилинишидан олдинги ва жорий қилингандан кейинги ҳолати натижалари таҳлили келтирилган.

РЕЗЮМЕ. В статье представлен анализ результатов работы системы электронного документооборота до и после внедрения науки и образования между высшими учебными заведениями, научно-инспекторскими институтами и ВАК в республике Узбекистан.

SUMMARY. The article presents an analysis of the results of the electronic document circulation system before and after the introduction of science and education between higher education institutions, scientific inspection institutes and the Higher Attestation Commission in the Republic of Uzbekistan.

SOLUTION OF PARABOLIC RADON TRANSFORM WITH GIVEN WEIGHT FUNCTION

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Tayanch so’zlar: integral geometriya masalasi, yagonalik teoremasi, Tixonov regulyarizatsiyasi, turg‘un algoritm, Fure almashtirishi.
Ключевые слова: задачи интегральной геометрии, теорема единственности, регуляризация Тихонова, устойчивый алгоритм, преобразование Фурье.
Key words: integral geometry problems, uniqueness theorem, Tikhonov regularization, stable algorithm, Fourier transform.

1. Statement of the problem of integral geometry and its connection with volterra integral equations

Consider the problem of finding a function $u(x,y)$ from the operator equation

$$\int_{\Gamma(x,y)} g(x,y,\xi,\eta)u(\xi,\eta)d\xi = f(x,y)\cdot \qquad (1)$$

where

$$\Gamma(x,y)=\left\{(\xi,\eta): y-\eta=(x-\xi)^2, -\infty<\xi<x,\ 0\le\eta\le y\le H\right\}$$

In [1], problems of integral geometry on families of parabolas with a weight function $g(x,y,\xi,\eta)=1$ are considered. An inversion formula for the problem of integral geometry on families of parabolas is obtained. Based on the idea of A.N. Tikhonov on the regularization of ill-posed problems, a sequence of approximate solutions of the exact solution of the problem is constructed.

In [2], the problem of integral geometry of Volterra type with respect to a family of parabolas with a weight function having a singularity is studied. Estimates of the stability of the solution of the problem in Sobolev spaces are obtained, which shows its weak ill-posedness, as well as the inversion formula.

In works [3-5], problems of integral geometry of spherical type were considered. Linear problems of integral geometry were considered in [6, 7].

In this paper, we consider problems of integral geometry on families of parabolas with a weight function $g(x,y,\xi,\eta)=\sqrt{y-\eta}$. An inversion formula for the problem of integral geometry on families of parabolas is obtained. Based on the idea of A.N. Tikhonov on the regularization of ill-posed problems, a sequence of approximate solutions of the exact solution of the problem is constructed.

2. Inversion formula of solution of the problem

Theorem. Let the function be $f(x,y)$ known in the strip $L_H=\left\{(x,y): x\in R^1, 0\le y\le H<\infty\right\}$, the weight function $g(x,y,\xi,\eta)=\sqrt{y-\eta}$.

Then the solution of equation (1) in the class of twice continuously differentiable finite functions supported in the strip is L_H unique and the representation takes place

$$\hat{u}(\lambda,y)=\frac{1}{i\lambda\pi}\frac{\partial}{\partial y}\int_0^y \frac{ch\left(\lambda\sqrt{y-\eta}\right)}{\sqrt{y-\eta}}\cdot\frac{\partial\hat{f}(\lambda,\eta)}{\partial\eta}d\eta\cdot$$

Proof. Equation (1) will be written, in the form

$$\int_0^y\left(u(x-h,\eta)-u(x+h,\eta)\right)d\eta=f(x,y),$$

where $h=\sqrt{y-\eta}$.

Now apply to both parts of equation (2) the Fourier transform with respect to the variable x. Taking into account the calculations at the beginning of the proof of the proposition, we write:

$$\int_0^y\hat{u}(\lambda,\eta)\frac{\lambda\cos\left(\lambda\sqrt{y-\eta}\right)}{\sqrt{y-\eta}}d\eta=\hat{\varphi}(\lambda,y)\cdot$$

Let us act on the last equation by the Volerra operator with the kernel

$$\frac{ch\left(\lambda\sqrt{y-\eta}\right)}{\sqrt{y-\eta}}$$

and apply the Fubini theorem to the left side of the resulting equation:

$$\int_0^y\frac{ch\left(\lambda\sqrt{y-\eta}\right)}{\sqrt{y-\eta}}\cdot\hat{\varphi}(\lambda,\eta)d\eta=$$

$$=\int_0^y\int_0^\eta\frac{ch\left(\lambda\sqrt{y-\eta}\right)}{\sqrt{y-\eta}}\cdot\hat{u}(\lambda,t)\times\frac{\lambda\cos\left(\lambda\sqrt{\eta-t}\right)}{\sqrt{\eta-t}}dtd\eta$$

$$=\lambda\int_0^y\int_0^\eta\frac{ch\left(\lambda\sqrt{y-\eta}\right)}{\sqrt{y-\eta}\sqrt{\eta-t}}\hat{u}(\lambda,t)\times\cos\left(\lambda\sqrt{\eta-t}\right)dtd\eta\cdot$$

We get the equation

$$\int_0^y\frac{ch\left(\lambda\sqrt{y-\eta}\right)}{\sqrt{y-\eta}}\cdot\hat{\varphi}(\lambda,\eta)d\eta=\lambda\int_0^y\hat{u}(\lambda,t)\cdot K_1(\lambda,y,t)dt, \qquad (3)$$

where

$$K_1(\lambda,y,t)=\int_0^1\frac{ch\left(\lambda\sqrt{y-t}\sqrt{1-s^2}\right)\cos\left(\lambda\sqrt{y-t}s\right)}{\sqrt{1-s^2}}ds\cdot$$

Given the expression for the table integral

$$\int_0^a\frac{ch\left(c\sqrt{a^2-x^2}\right)}{\sqrt{a^2-x^2}}\cos bxdx=\frac{\pi}{2}, \text{ at } b=c>0,\ a>0,$$

we obtain that the kernel K_1

$$\int_0^1 \frac{ch\left(\lambda\sqrt{y^2-t^2}\sqrt{1-s^2}\right)\cos\left(\lambda\sqrt{y^2-t^2}s\right)}{\sqrt{1-s^2}}ds=\frac{\pi}{2}.$$

Taking into account this equality, we rewrite (3) in the form

$$\pi\lambda\int_0^y \hat{u}(\lambda,\eta)d\eta=\int_0^y \frac{ch\left(\lambda\sqrt{y-\eta}\right)}{\sqrt{y-\eta}}\cdot\hat{\varphi}(\lambda,\eta)d\eta.$$

Differentiating both parts of the last equation, we arrive at the expression:

$$\hat{u}(\lambda,y)=\frac{1}{i\lambda\pi}\frac{\partial}{\partial y}\int_0^y \frac{ch\left(\lambda\sqrt{y-\eta}\right)}{\sqrt{y-\eta}}\cdot\frac{\partial\hat{f}(\lambda,\eta)}{\partial\eta}d\eta.$$

In the last inversion formula, one can notice that for a fixed $0\le\eta\le y\le H$ $ch\left(\lambda\sqrt{y-\eta}\right)$ tend to infinity as $|\lambda|\to\infty$. Based on the idea of A. N. Tikhonov [8] on the regularization of ill-posed problems, we construct a sequence of approximate solutions to the exact solution of problem (1).

We introduce a sequence of approximate solutions as follows

$$\hat{u}_\alpha(\lambda,y)=e^{-\alpha^2\lambda^2}\hat{u}(\lambda,y),$$

where $\alpha>0-$is the regularization parameter.

Note that the inverse Fourier transform of $e^{-\alpha^2\lambda^2}$ expressed as follows

$$W_\alpha(x)=\int_{-\infty}^{+\infty} e^{-\alpha^2\lambda^2}\cdot e^{-i\lambda x}d\lambda=\frac{1}{2\alpha\sqrt{\pi}}e^{-\frac{x^2}{4\alpha^2}}. \quad (4)$$

Let us calculate the inverse Fourier transform of the function $\hat{u}_\alpha(\lambda,y)$

$$F^{-1}\left[\hat{u}_\alpha(\lambda,y)\right]=u_\alpha(x,y), \quad (5)$$

this is a normal function expressed as a convolution

$$u_\alpha(x,y)=\int_{-\infty}^{+\infty} W_\alpha(x-\xi)u(\xi,y)d\xi. \quad (6)$$

Denote

$$R_\alpha(x,h)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-\alpha^2\lambda^2}ch(\lambda h)d\lambda, \quad (7)$$

where $h=\sqrt{y-\eta}$.

Then from (4), (5), (6) and (7) we obtain

$$u_\alpha(x,y)=-\frac{1}{2\alpha\sqrt{\pi^3}}\frac{\partial}{\partial y}\int_{-\infty}^{x}\int_0^{y}\int_{-\infty}^{+\infty} e^{\frac{y-\eta-(\xi-\mu)^2}{4\alpha^2}}\cos\frac{(\xi-\mu)\sqrt{y-\eta}}{2\alpha^2}\frac{1}{\sqrt{y-\eta}}\frac{\partial f(\mu,\eta)}{\partial\eta}d\mu d\eta d\xi$$

3. Numerical experiment

In we conduct a uniform grid in a rectangular area $D=[-a;a]\times[c;d]$.

Let us divide the segment $[-a;a]$ into axes Ox and $[c;d]$ axes Oy into n_x-1 and n_y-1 parts, respectively. $x_i=-a+(i-1)h_x,\ y_j=c+(j-1)h_y$.

We find approximations of functions.

$$u_\alpha(x_i,y_j)=-\frac{G_{ij+1}-G_{ij-1}}{4\alpha h_y\sqrt{\pi^3}},$$

where

$$G_{ij}=\int_{-a}^{x_i}\int_0^{y_j}\int_{-a}^{+a} e^{\frac{y_j-\eta-(\xi_i-\mu)^2}{4\alpha^2}}\cos\frac{(\xi_i-\mu)\sqrt{y_j-\eta}}{2\alpha^2}\frac{\partial f(\xi,\eta)}{\partial\eta}\frac{d\mu}{\sqrt{y_j-\eta}}d\eta d\xi,$$

where $\alpha-$is regularization parameter.

To choose the regularization parameter α, we construct the dependence of the relative error of the solution $u_\alpha(x,y)$ on the exact solution $u(x,y)$ [9].

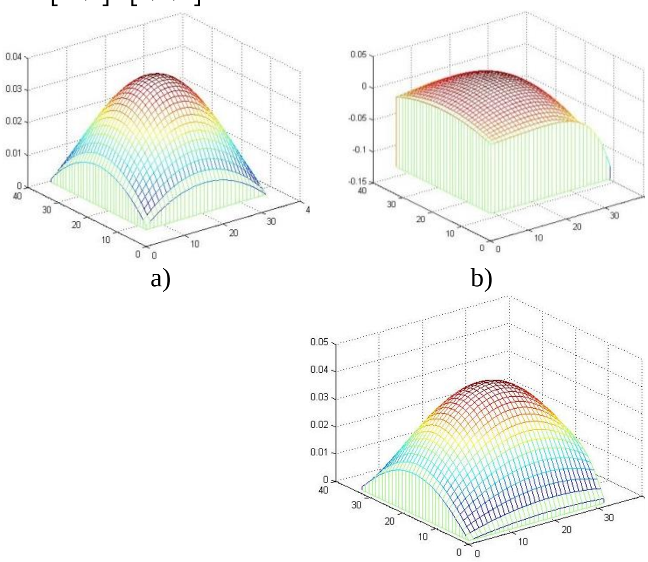
$$\sigma(\alpha)=\frac{\left\|u_\alpha(x,y)-u(x,y)\right\|_{L_2}}{\left\|u(x,y)\right\|_{L_2}}$$

We choose a value α_{opt} that reaches minimizes the last equation

As a test case, consider the following function

$$f(x,y)=\frac{16xy\sqrt{y}}{15}(1-2y)$$

The calculation results for $n_x=n_y=41$ in the grid $D=[-1;1]\times[0;0,5]$ are presented in Fig.1.



a) b) c)

Fig.1 a) Exact solution b) recovery at $\alpha=0{,}1021$ c) **recovery at** $\alpha=0{,}2174$

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REZYUME. Ushbu maqolada parabolalar oilasida vazn funksiyasi bilan berilgan integral geometriya masalasi qaraladi. Birinchi oʻzgaruvchi boʻyicha masalaning Fure obrazi topilgan. Tixonov regulyarizatsiyasi asosida masala aniq echimiga yaqin echimlar ketma-ketligiga qurilgan.

РЕЗЮМЕ. В данной работе рассматривается задача интегральной геометрии на семействе параболического типа с весовой функцией. Получена формула обращения образа Фурье по первой переменной. Последовательность приближенных решений строится на основе регуляризации Тихонова.

SUMMARY. In this paper we consider the problem of integral geometry on a family of parabolic type with a weight function. An inversion formula for the Fourier image with respect to the first variable is obtained. A sequence of approximate solutions is constructed based on Tikhonov's regularization.

KVADRAT TEŃLEMEGE KELTIRIW ARQALI SHESHILETUǴIN QURAMALI ALGEBRALIQ TEŃLEMELERDI SHESHIW USILLARI

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Tayansh soʻzlar: kvadrat tenglamalar, murakkab tenglamalar, tenglamalarni yechish usuli, misol.
Ключевые слова: квадратные уравнения, сложные уравнения, метод решения уравнений, пример.
Key words: quadratic equations, complex equations, method of solving equations, example.

Matematikada ayırım quramalı teńlemeler túrlendiriwler hám hár qıylı usıllar arqalı kvadrat teńlemelerge keltirilip sheshiledi.

1-mısal. Teńlemeni sheshiń

$$(x+1)(x+2)(x+3)(x+4)=1680$$

Sheshiw. Berilgen teńlemeni tómendegishe túrlendiremiz:

$$(x+1)(x+4)(x+2)(x+3)=1680 \Leftrightarrow \left(x^2+5x+4\right)\left(x^2+5x+6\right)=1680 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x^2+5x+5=y \\ (y-1)(y+1)=1680 \end{cases} \Leftrightarrow \begin{cases} x^2+5x+5=y \\ y^2-1=1680 \end{cases} \Leftrightarrow \begin{cases} x^2+5x+5=y \\ y^2=1681 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x^2+5x+5=y \\ y_{1,2}=\pm 41 \end{cases}$$

1. $x^2+5x+5=y_1 \Leftrightarrow x^2+5x+5=41 \Leftrightarrow x^2+5x-36=0 \Leftrightarrow \left[\begin{array}{l} x_1=-9 \\ x_2=4 \end{array}\right.$

2. $x^2+5x+5=y_2 \Leftrightarrow x^2+5x+5=-41 \Leftrightarrow x^2+5x+46=0 \Leftrightarrow \left[\begin{array}{l} D<0 \\ x=\varnothing \end{array}\right.$

Demek, berilgen teńleme eki sheshimge iye.

Juwabı: $x_1=-9,\ x_2=4.$

2-mısal. Teńlemeni sheshiń

$$(x+5)^4-13(x+5)^2x^2+36x^4=0$$

Sheshiw. Berilgen teńlemeni tolıq kvadrat ajıratıw usılı arqalı sheshemiz, onıń ushın teńlemeniń eki jaǵına $25(x+5)^2x^2$ ańlatpasın qosıp hám alamız:

$$(x+5)^4-13(x+5)^2x^2+36x^4=0 \Leftrightarrow (x+5)^4+25(x+5)^2x^2+36x^4-$$
$$-13(x+5)^2x^2=25(x+5)^2x^2 \Leftrightarrow (x+5)^4+12(x+5)^2x^2+36x^4=$$
$$=25(x+5)^2x^2 \Leftrightarrow \left((x+5)^2+6x^2\right)^2-\left(5(x+5)x\right)^2=0 \Leftrightarrow$$
$$\Leftrightarrow \left((x+5)^2+6x^2-5(x+5)x\right)\left((x+5)^2+6x^2+5(x+5)x\right)=0 \Leftrightarrow$$
$$\Leftrightarrow \left[\begin{array}{l} (x+5)^2+6x^2-5(x+5)x=0 \\ (x+5)^2+6x^2+5(x+5)x=0 \end{array}\right.$$

1.

$$(x+5)^2+6x^2-5(x+5)x=0 \Leftrightarrow x^2+10x+25+6x^2-5x^2-25x=0 \Leftrightarrow$$
$$\Leftrightarrow 2x^2-15x+25=0 \Leftrightarrow \left[\begin{array}{l} x_1=\dfrac{5}{2} \\ x_2=5 \end{array}\right.$$

2.

$$(x+5)^2+6x^2+5(x+5)x=0 \Leftrightarrow x^2+10x+25+6x^2+5x^2+25x=0 \Leftrightarrow$$
$$\Leftrightarrow 12x^2+35x+25=0 \Leftrightarrow \left[\begin{array}{l} x_3=-\dfrac{5}{4} \\ x_4=-\dfrac{5}{3} \end{array}\right.$$

Juwabı: $x_1=\dfrac{5}{2},\ x_2=5,\ x_3=-\dfrac{5}{4},\ x_4=-\dfrac{5}{3}.$

3-mısal. Teńlemeni sheshiń

$$2\left(x^2-x+1\right)^2=x^2\left(8x^2-5x+5\right)$$

Sheshiw. Berilgen teńlemeni tómendegishe túrlendiremiz:

$$2\left(x^2-x+1\right)^2=x^2\left(8x^2-5x+5\right) \Leftrightarrow 2\left(x^2-x+1\right)^2=$$
$$=x^2\left(5x^2-5x+5+3x^2\right) \Leftrightarrow$$
$$\Leftrightarrow 2\left(x^2-x+1\right)^2=x^2\left(5x^2-5x+5\right)+3x^4 \Leftrightarrow$$
$$\Leftrightarrow 2\left(x^2-x+1\right)^2=5x^2\left(x^2-x+1\right)+3x^4$$

$$2\left(x^2-x+1\right)^2=5x^2\left(x^2-x+1\right)+3x^4 \qquad (1)$$

(1) teńlemeniń eki jaǵın $x^2\left(x^2-x+1\right)\neq 0$ ańlatpasına bólemiz:

$$\frac{2\left(x^2-x+1\right)}{x}=5+\frac{3x^2}{x^2-x+1} \Leftrightarrow \begin{cases} \dfrac{x^2-x+1}{x^2}=y \\ 2y=5+\dfrac{3}{y} \end{cases} \Leftrightarrow \begin{cases} \dfrac{x^2-x+1}{x^2}=y \\ 2y^2-5y+3=0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \dfrac{x^2-x+1}{x^2}=y \\ \left[\begin{array}{l} y_1=-\dfrac{1}{2} \\ y_2=3 \end{array}\right. \end{cases}$$

1. $\dfrac{x^2-x+1}{x^2}=y_1 \Leftrightarrow \dfrac{x^2-x+1}{x^2}=-\dfrac{1}{2} \Leftrightarrow \begin{cases} 2x^2-2x+2=-x^2 \\ x\neq 0 \end{cases} \Leftrightarrow$

$$\Leftrightarrow 3x^2-2x+2=0 \Leftrightarrow \left[\begin{array}{l} D<0 \\ x=\varnothing \end{array}\right.$$