ABOUT ONE OF THE PROBLEMS OF INTEGRAL GEOMETRY IN A STRIP ON A FAMILY OF BROKEN LINES

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Ключевые слова: задача интегральной геометрии, обратные задачи, некорректные задачи, численное решение, преобразование Фурье.
Key words: problem of integral geometry, inverse tasks, ill-posed tasks, numerical solution, Fourier transformation.

In the article we consider the definition of the internal structure of the object by their integral data on a family of broken lines. An analytic representation of the solution in the class of smooth finite functions was obtained. On the basis of the inversion formula, the internal structure of the object is restored using integral characteristics. Estimates of the solution of the problem in Sobolev spaces are presented, which implies its weak incorrectness. Similar problems were considered in the articles (Begmatov, Djaykov pp.6, Begmatov, Djaykov pp.14, Begmatov, Djaykov pp. 478). Uniqueness theorems, stability estimates, and inversion formulas for certain classes of linear problems were obtained in work (Begmatov and others pp. 5).

Obtaining images of the internal anatomical structure of human organs and tissues, the study of their functions in norm and in pathology are of great interest for medical science. Diagnosis of diseases and their treatment, analysis of the results of treatment should be based on data obtained by medical visualization (Charles L. Epstein pp. 21).

The problem of early differential diagnosis and treatment planning for various diseases is urgent, which requires imaging techniques such as single-photon emission computed tomography (SPECT) and positron emission computed tomography (PECT) as well as a combination of tomographs in one setting imaging these methods (PET / CT, SPECT / CT) (Fedorov pp. 124).

We introduce the notation, which we shall use later:

$$(x,y)\in R^2,\quad (\xi,\eta)\in R^2,\quad \lambda\in R^1,\quad \mu\in R^1,$$

$$S=\{(x,y):\ x\in R^1,\ y\in[0,H],\ H<\infty\}.$$

In the strip S we investigate families of polygonal lines, which are defined by relations

$$\Upsilon(x,y)=\{(\xi,\eta):\ x-\xi=y-\eta,\ 0\le y\le H\}.$$

$\mathrm{P}(x,y)-$ region of bounded curve $\Upsilon(x,y)$ and $\eta=0$.

Statement of the problem. Recover the function of two variables $u(x,y)$, if in the strip S there are known integrals of it along the curves of the family $\{\Upsilon(x,y)\}$ with weight function $g(x,y)$:

$$\int\limits_{\Upsilon(x,y)} g(x,\xi)u(\xi,\eta)ds = f(x,y). \qquad (1)$$

THE PROBLEM OF INTEGRAL GEOMETRY WITH WEIGHT FUNCTION OF GENERAL FORM

Theorem 1. Let the function $f(x,y)$ is known for all $(x,y)\in S$, the weight function has the form $g(x,\xi)=e^{-k(x-\xi)}$.

Then the solution of the equation 1 in the class $C_0^{k+1}(S)$ is unique and is expressed through the function $f(x,y)$ according to the formula

$$u(x,y)=\frac{1}{k!}\left(\frac{\partial}{\partial y}+\frac{\partial}{\partial x}\right)^{k+1} f(x,y)$$

and satisfies the inequality

$$\|u(x,y)\|_{L_2}\le C_1\|f(x,y)\|_{W_2^{k+1}}$$

where C_1- is a constant.

Proof. The equation (1) can be represented in the following form

$$\int\limits_0^y u(x-h,\eta)(y-\eta)^k d\eta = f(x,y), \qquad (3)$$

where $h=y-\eta$.

Let's apply the Fourier transform with respect to the first variable to both sides of equation (3):

$$\hat f(\lambda,y)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{+\infty} e^{i\lambda x}\int\limits_0^y u(x-h,\eta)(y-\eta)^k d\eta dx=\int\limits_0^y \hat u(\lambda,\eta)(y-\eta)^k e^{i\lambda h}d\eta$$

We obtain the equation:

$$\int\limits_0^y \hat u(\lambda,\eta)(y-\eta)^k e^{i\lambda(y-\eta)}d\eta=\hat f(\lambda,y), \qquad (4)$$

We apply the Laplace transform with respect to the variable y to the equation (4):

$$\int\limits_0^\infty e^{-py}\hat f(\lambda,y)dy=\int\limits_0^\infty e^{-py}\int\limits_0^y \hat u(\lambda,\eta)(y-\eta)^k e^{i\lambda(y-\eta)}d\eta dy \qquad (5)$$

Thus, from the equation (5) we obtain:

$$\tilde{\hat u}(\lambda,p)\cdot I(\lambda,p)=\tilde{\hat f}(\lambda,p), \qquad (6)$$

where $\tilde{\hat u}(\lambda,p)=\int\limits_0^\infty e^{-p\eta}\hat u(\lambda,\eta)d\eta$,

$$\tilde{\hat f}(\lambda,p)=\int\limits_0^\infty e^{-py}\hat f(\lambda,y)dy.$$

$$I(\lambda,p)=\frac{k!}{(p-i\lambda)^{k+1}} \qquad (7)$$

from (6) we obtain

$$\tilde{\hat u}(\lambda,p)=\frac{(p-i\lambda)^{k+1}}{k!}\tilde{\hat f}(\lambda,p). \qquad (8)$$

We apply the inverse Laplace transform with respect p and the inverse Fourier transform with respect λ to both sides of equation (8). Taking into account the properties of the Laplace and Fourier transforms, we obtain the inversion formula

$$u(x,y)=\frac{1}{k!}\left(\frac{\partial}{\partial y}+\frac{\partial}{\partial x}\right)^{k+1} f(x,y). \qquad (9)$$

Using the properties of the Laplace and Fourier transforms, and also taking into account (9), we obtain the estimate

$$\left\| u(x,y) \right\|_{L_2} \le C_1 \left\| f(x,y) \right\|_{W_2^{k+1}},$$

where $C_1 -$ is a constant.

Theorem 2. Let the function $f(x,y)$ is known for all $(x,y) \in S$, and also satisfy the following conditions:

1. $f(x,y)$ is finite on the variable x;
2. $f(x,y)$ has continuous $k+1$ order partial derivatives
3. $f(x,y)$ together with its partial derivatives up to the $k+1$ order inclusive, turns to 0 on the boundaries of the strip S, i.e. at y = 0 and y = H.

Then there exists a solution of Problem 1 in the class of twice continuously differentiable functions that are finite in the argument x, defined by the formula (9).

Proof. From the conditions imposed on the function $f(x,y)$ it is clear that to both sides of (9) we can apply the Fourier transform with respect to the variable x and the Laplace transform with respect to the variable y. Using the Fourier and Laplace transform properties, we obtain

$$\tilde{\hat{u}}(\lambda,p) = \frac{(p-i\lambda)^{k+1}}{k!}\tilde{\hat{f}}(\lambda,p),$$

or

$$\tilde{\hat{f}}(\lambda,p) = \frac{k!}{(p-i\lambda)^{k+1}}\tilde{\hat{u}}(\lambda,p).$$

Using formula (7), we can obtain

$$\tilde{\hat{f}}(\lambda,p) = I(\lambda,p)\cdot\tilde{\hat{u}}(\lambda,p), \qquad (10)$$

Where $I(\lambda,p) = \dfrac{k!}{(p-i\lambda)^{k+1}}$.

Applying the inverse Laplace transform with respect to the variable p to (10), we obtain the following expression

$$\hat{f}(\lambda,y) = \int_0^y \hat{u}(\lambda,\eta)(y-\eta)^k e^{i\lambda(y-\eta)}d\eta.$$

Applying the inverse Fourier transform with respect to x to the last equation, we get:

$$f(x,y) = \int_0^y u(x-h,\eta)(y-\eta)^k d\eta.$$

NUMERICAL IMPLEMENTATION

We define the Radon transform on the family of broken lines as follows:

$$f(x,y) = \int_{\Upsilon(x,y)} u(\xi,\eta)ds1 \qquad (11)$$

where $\Upsilon(x,y) = \{(\xi,\eta): x-\xi = y-\eta,\ 0 \le y \le H\}$.

The theoretical results obtained in this part of our work are investigated by experimental studies. For the experiments there was compiled the software using the C # program.

We introduce a uniform grid in the rectangular region $D = [a,b]\times[c,d]$. We seek approximate solutions of the problem on this rectangle.

We obtain the functions $f(x,y)$ from the formula (2). In the arrays of the values of $f(x_i,y_j) = f_{ij}$, we used definitions approximating the values of the function u_{ij}^A and comparing's with the analytic values of u_{ij}^E.

We rewrite the obtained inversion formula in the form

$$u(x,y) = \frac{\partial}{\partial x}f(x,y) + \frac{\partial}{\partial y}f(x,y).$$

We find approximate solutions of the problem on the rectangle D.

The scheme of the algorithm for solving the problem is as follows:

Step1. We divide the segment $[a,b]$ on the axis Ox and $[c,d]$ on the axis Oy into $n_x - 1$ and $n_y - 1$ parts respectively. $x_i = a + (i-1)h_x$, $y_j = b + (j-1)h_y$

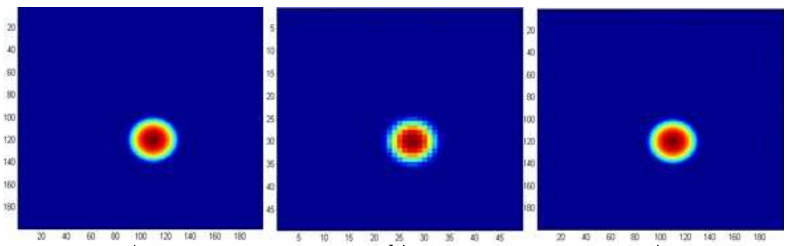
Step2. Approximations of functions $u(x_i,y_j)$ we will denote by u_{ij}^A.

$$u_{ij}^A = \frac{f_{ij+1} - f_{ij-1}}{2h_y} + \frac{f_{i+1j} - f_{i-1j}}{2h_x} \qquad (9)$$

Example 1. Let we give the mathematic phantom

$$u(x,y) = \begin{cases} e^{-\frac{r^2}{r^2 - \left((x-x_0)^2 + (y-y_0)^2\right)}}, & (x-x_0)^2 + (y-y_0)^2 < r^2, \\ 0, & (x-x_0)^2 + (y-y_0)^2 > r^2. \end{cases}$$

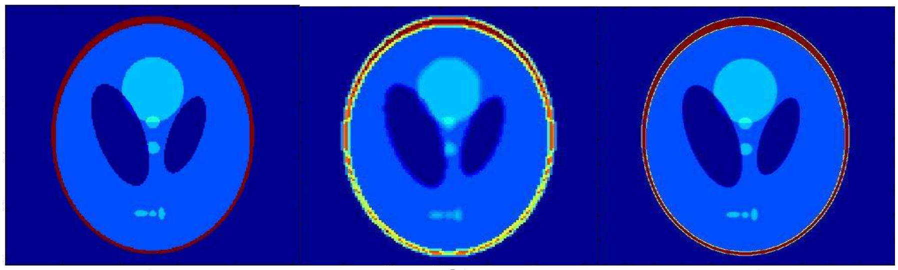
Figure 1 shows the results of restoration.



a) b) c)

Figure 1. a) Original phantom, b) Recovery with $N = 51$, c) Recovery with $N = 201$.

Example 2. The second numerical experiment was aimed at comparing the two phantoms of the original and the reconstructed Shepp-Logan phantom. Figure 2 shows the results of restoration.



a) b) c)

Figure.2 Restoration of the Shepp-Logan phantom a) Original Shepp-Logan phantom,
b) Recovery with $N = 128$, c) Recovery with $N = 512$.

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РЕЗЮМЕ

Мақолада вазн функцияси билан берилган интеграл геометрия масаласи ўрганилади. Ечимнинг ягоналиги ва мавжудлиги исботланган ва силлиқ финит функциялар синфида ечимнинг аналитик кўриниши олинган. Соболев фазосида масала ечимининг баҳоси топилган бўлиб, бундан унинг кучсиз корректмаслиги келиб чиқади. Мақоланинг иккинчи қисмида олинган назарий маълумотлар экспериментал берилганлар билан тадқиқ қилинади. Масалани ечиш учун қурилган алгоритмнинг сонли ва график натижалари келтирилган. Бундай масалалар сейсмик тадқиқотлар, геофизик ва аэрокосмик кузатувларни талқин қилишда, астрофизика ва гидроакустиканинг тескари масалаларини ечишда амалий тадқиқотлар учун кўплаб иловаларига эга.

РЕЗЮМЕ

В статье исследуется задача интегральной геометрии в полосе на семействе отрезков с заданной весовой функцией общего вида. Доказана теорема единственности и теорема существования решения задачи, получено аналитическое представление решения в классе гладких конечных функций. Представлена оценка решения задачи в пространствах Соболева, из которой вытекает её слабая некорректность. Полученные теоретические результаты исследуются по экспериментальным данным. Приведены численные и графические результаты применения этих алгоритмов для решения задачи. Такие задачи имеют многочисленные приложения в математическом изучении проблем сейсморазведки, интерпретации геофизических и аэрокосмических наблюдений, в решении обратных задач астрофизики и гидроакустики.

SUMMARY

The article deals with the study of problems of integral geometry in a strip on a family of broken lines with a given weight function of a general form. The theorems of uniqueness and existence for the solution of the task are proved; an analytic representation of the solution in the class of smooth finite functions is obtained. An estimate of the solution of the task in Sobolev spaces is presented, from which its weak incorrectness follows. The obtained theoretical results are investigated by experimental data. The numerical and graphical results of applying these algorithms to the solution of the task are given. Such task have numerous applications in the mathematical study of the problems of seismic prospecting, the interpretation of geophysical and aerospace observations, in solving inverse task of astrophysics and hydro acoustics.

Ximiya, Geografiya, Biologiya, Zoologiya

KOMPYUTER TEXNOLOGIYALARÍ JÁRDEMINDE XIMIYA PÁNINIŃ TIYKARǴÍ TÚSINIKLERI

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Tayanch so’zlar: o’qitish, uslublar, kompyuterlashtirish, o’quv jarayoni, zamonaviy kompyuter texnologiyalari, zamonaviylashtirish.

Ключевые слова: обучение, методы, компьютеризация, воспитательный процесс, современные компьютерные технологий, модернизация.

Key words: training, methods, computerization, educational process, modern computer technology, modernization.

Insan rawajlanıwınıń házirgi basqıshı ústin turatuǵın jónelis sıpatında adamgershilik (shaxsqa baǵdarlanǵan) tálim paradigmasın qoyadı, bul bolsa insannıń individual pázıyletlerin maksimal dárejede rawajlandırıwdıń hám onnan paydalanıw uluwma jámiyet párawanlıǵın támiyinleydi. Gumanistik paradigma barlıq rawajlanǵan mámleketlerdiń tálim sistemasınıń orayında jaylasqan. Shaxstıń hár tárepleme hám tolıq rawajlanıwı ishki tálimde tiykarǵı wazıypa sıpatında qoyılǵan.

Islep shıǵılǵan kompyuter programmalarınan sabaqlarda shınıǵıwlardı shólkemlestiriwdiń klastaǵı forması sıpatında paydalanıwdı usınıs etemiz. Biraq sonı atap ótiw kerek, olardan individual ózbetinshe jumıslardı shólkemlestiriwde hám aralıqtan oqıwda nátiyjeli paydalanıw múmkin. Ximiya sabaǵın kompyuter menen támiyinlewdi shólkemlestiriw ximiya pániniń ayriqsha qásiyetleri hám kompyuter klasındaǵı jumıslardıń sanı sheklewleri menen baylanıslı bir qatar ayrıqshalıqlarǵa iye. Qaǵıydaǵa kóre, kompyuter klası 15 dana kompyuterden ibarat bolıp, ximiyalıq tájiriybe ótkeriw ushın arnawlı úskeneler menen úskenelenbegen. Nátiyjede, biz tańlaw principleri tiykarında saylanǵan bir saatlıq temalar ushın kompyuter járdemin usınıs etemiz.

Kompyuter programmaları járdeminde alıp barılatuǵın sabaqlar kombinaciyalıq xarakterge iye hám kompyuter